What can we learn about the neutron-proton effective mass splitting from constraints on the density dependence of nuclear symmetry energy around normal density?

Bao-An Li[1, 2, *] and Xiao Han[2]

[1]Department of Physics and Astronomy, Texas A&M University-Commerce, Commerce, TX 75429-3011, USA
[2]Department of Applied Physics, Xi'an Jiaotong University, Xi'an 710049, China

According to the Hugenholtz-Van Hove theorem, nuclear symmetry energy $E_{sym}(\rho)$ and its slope $L(\rho)$ at an arbitrary density ρ are determined by the nucleon isovector (symmetry) potential $U_{sym}(\rho, k)$ and its momentum dependence $\frac{\partial U_{sym}}{\partial k}$. The latter determines uniquely the neutron-proton effective k-mass splitting $m^*_{n-p}(\rho, \delta) \equiv (m^*_{\rm n} - m^*_{\rm p})/m$ in neutron-rich nucleonic matter of isospin asymmetry δ. Using currently available constraints on the $E_{sym}(\rho_0)$ and ł0 at normal density ρ_0 of nuclear matter from 24 recent analyses of various terrestrial nuclear laboratory experiments and astrophysical observations, we infer the corresponding neutron-proton effective k-mass splitting $m^*_{n-p}(\rho_0, \delta)$. While the mid-values of the $m^*_{n-p}(\rho_0, \delta)$ obtained from most of the studies are remarkably consistent with each other and scatter very closely around an empirical value of $m^*_{n-p}(\rho_0, \delta) = 0.24 \cdot \delta$, reduced experimental error bars and more complete information about the uncertainties in extracting the $E_{sym}(\rho_0)$ and ł0 from data using various models are much needed in order to access more accurately how reliably the inferred $m^*_{n-p}(\rho_0, \delta)$ can be used in investigating many interesting issues in both nuclear physics and astrophysics.

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I. INTRODUCTION

The ultimate goal of investigating properties of neutron-rich nucleonic matter through terrestrial nuclear laboratory experiments and astrophysical observations is to understand the underlying isospin dependence of strong interaction in nuclear medium [1]. The Equation of State (EOS) of neutron-rich nucleonic matter can be written within the parabolic approximation in terms of the binding energy per nucleon at density ρ as $E(\rho, \delta) = E(\rho, \delta = 0) + E_{\rm sym}(\rho)\delta^2 + \mathcal{O}(\delta^4)$ where $\delta \equiv (\rho_n - \rho_p)/(\rho_p + \rho_n)$ is the neutron-proton asymmetry and $E_{\rm sym}(\rho)$ is the density-dependent nuclear symmetry energy. The latter has important applications in many areas of both nuclear physics, see, e.g., refs. [2–8] and astrophysics, see, e.g., refs. [9–11]. However, the density dependence of nuclear symmetry energy has been among the most uncertain properties of neutron-rich nucleonic matter. Predictions using various many-body theories and interactions diverge quite broadly especially at abnormal densities. It is thus exciting to see that significant progress has been made recently in constraining the $E_{sym}(\rho)$ around ρ_0, see, e.g., ref. [12] based on model analyses of experimental and/or observational data. In particular, at least 24 recent studies have extracted the slope $L(\rho_0) \equiv [3\rho(\partial E_{\rm sym}/\partial\rho]_{\rho_0}$ and $E_{\rm sym}(\rho_0)$ at ρ_0 [13–39]. It is thus interesting to ask timely what we can learn about the isospin dependence of in-medium nuclear interaction from the extracted constrains on ł0 and $E_{sym}(\rho_0)$. In this work, we answer this question at the mean-field level by using a formulism developed earlier based on the Hugenholtz-Van Hove (HVH) theorem. In particular, we infer both the magnitude of the symmetry potential $U_{\rm sym}(\rho_0, k_F)$ and the neutron-proton effective k-mass splitting $m^*_{n-p}(\rho_0, \delta)$ corresponding to each of the 24 constraints on $E_{sym}(\rho_0)$ and ł0 at ρ_0. The consistency of the extracted values for $U_{\rm sym}(\rho_0, k_F)$ and $m^*_{n-p}(\rho_0, \delta)$ from various constraints is then examined. It is found that while the mean values of the $U_{\rm sym}(\rho_0, k_F)$ and $m^*_{n-p}(\rho_0, \delta)$ from different studies are consistent with each other and most of them scatter closely around $U_{sym}(\rho_0, k_F)$ =29 MeV and $m^*_{n-p}(\rho_0, \delta) = 0.24 \cdot \delta$, respectively, the individual uncertainties from different analyses are still large. Reducing the experimental error bars and better understanding the uncertainties of the model analyses are much needed in order to use reliably the extracted mean values of the $U_{\rm sym}(\rho_0, k_F)$ and $m^*_{n-p}(\rho_0, \delta)$ in solving many important problems in both nuclear physics and astrophysics.

*Electronic address: Bao-An.Li@tamuc.edu

II. RELATIONSHIP BETWEEN NEUTRON-PROTON EFFECTIVE MASS SPLITTING AND SYMMETRY ENERGY BASED ON THE HUGENHOLTZ-VAN HOVE THEOREM

According to the well-known Lane potential [40], the neutron/proton (n/p) single-particle potential $U_{n/p}(\rho, k, \delta)$ can be well approximated by

$$U_{\mathrm{n/p}}(\rho, k, \delta) \approx U_0(\rho, k) \pm U_{\mathrm{sym}}(\rho, k)\delta \ , \tag{1}$$

where the $U_0(\rho, k)$ and $U_{\mathrm{sym}}(\rho, k)$ are, respectively, the nucleon isoscalar and isovector (symmetry) potentials for nucleons with momentum k in asymmetric nuclear matter of isospin asymmetry δ at density ρ. Their momentum dependence is normally characterized by the nucleon effective k-mass

$$m^*_\tau/m = [1 + \frac{m}{\hbar^2 k_F}\frac{dU_\tau}{dk}|_{k_F}]^{-1} \tag{2}$$

where τ =n, p and 0 for neutrons, protons and nucleons, respectively, and $m = (m_n + m_p)/2$ is the average mass of nucleons in free-space. While the nucleon isoscalar potential and its momentum dependence, especially at ρ_0, have been relatively well determined, our knowledge about the isovector potential $U_{sym}(\rho, k)$ and its momentum dependence $\frac{\partial U_{sym}}{\partial k}$ even at normal density is still very poor. However, from the structure of rare isotopes and mechanism of heavy-ion reactions to properties of neutron stars, solutions to many interesting issues depend critically on the nucleon isovector potential and its momentum dependence.

Using the Brueckner theory [41] or the Hugenholtz-Van Hove (HVH) theorem [42], the $E_{sym}(\rho)$ and $L(\rho)$ can be expressed as [28, 43–45]

$$E_{\mathrm{sym}}(\rho) = \frac{1}{3}\frac{\hbar^2 k_F^2}{2m_0^*} + \frac{1}{2}U_{\mathrm{sym}}(\rho, k_F), \tag{3}$$

$$L(\rho) = \frac{2}{3}\frac{\hbar^2 k_F^2}{2m_0^*} + \frac{3}{2}U_{\mathrm{sym}}(\rho, k_F) + \frac{\partial U_{\mathrm{sym}}}{\partial k}|_{k_F} k_F, \tag{4}$$

where $k_F = (3\pi^2\rho/2)^{1/3}$ is the nucleon Fermi momentum. We emphasize that these relationships are general and independent of the many-body theory and/or interaction used to calculate the $U_{sym}(\rho, k)$ and m_0^*. It is well known that given both two-body and three-body nuclear interactions, the resulting nucleon potential often depends on the many-body theory used. On the other hand, the single-particle mean-field potential is often the one directly tested in comparing model calculations with experimental/observational data. For example, it is the input for most shell model calculations of nuclear structure and transport model simulations of nuclear reactions. The above expressions for $E_{sym}(\rho)$ and $L(\rho)$ indicate that one can use the density dependence of nuclear symmetry energy extracted from experiments/observations to test directly the nuclear isovector potential and its momentum dependence, or vice versa, without the hinderance of remaining difficulties and uncertainties in nuclear many-body theories. Here, we are interested in learning about the isospin dependence of in-medium nuclear interaction at ρ_0 from the constrained $E_{sym}(\rho_0)$ and ł0. More explicitly,

$$U_{\mathrm{sym}}(\rho_0, k_F) = 2\left[E_{\mathrm{sym}}(\rho_0) - \frac{1}{3}\frac{m}{m_0^*}E_F(\rho_0)\right], \tag{5}$$

$$\left.\frac{dU_{\mathrm{sym}}}{dk}\right|_{k_F}(\rho_0) = \left[L(\rho_0) - 3E_{\mathrm{sym}}(\rho_0) + \frac{1}{3}\frac{m}{m_0^*}E_F(\rho_0)\right] \Big/ k_F, \tag{6}$$

where $E_F(\rho_0)$ is the Fermi energy at ρ_0. While the $U_{sym}(\rho_0, k_F)$ is completely determined by $E_{sym}(\rho_0)$ and $\frac{m}{m_0^*}$, the $\left.\frac{dU_{\mathrm{sym}}}{dk}\right|_{k_F}(\rho_0)$ also depends on ł0.

The nucleon effective mass describes to leading order effects related to the non-locality of the underlying nuclear interactions and the Pauli exchange effects in many-fermion systems [46–48]. While the nucleon isoscalar effective k-mass is well determined to be $m_0^*/m = 0.7 \pm 0.05$ at ρ_0 [47], essentially nothing is known about the nucleon isovector effective mass [48]. Knowledge about the neutron-proton effective mass splitting is essential for understanding many interesting questions in both nuclear physics and astrophysics [10, 49–54], such as, pairing and superfluidity in nuclei, properties of rare isotopes, isospin transport in heavy-ion reactions, thermal and transport properties of neutron star crust and cooling mechanism of protoneutron stars. Unfortunately, even the sign of the neutron-proton effective mass splitting, not to mention its magnitude, has been a longstanding and controversial issue. While some theories predict that $m^*_{\mathrm{n}} \geq m^*_{\mathrm{p}}$, the opposite has often been shown by studies using different models or interactions, see, e.g., refs.

TABLE I: Constrained values of $E_{sym}(\rho_0)$ and ł0 from analyses of terrestrial nuclear experiments and astrophysical observations

Analysis	$E_{sym}(\rho_0)$	ł0	Ref.
Atomic masses and n-skin of Sn isotopes (2011)	30.5 ± 3	52.5 ± 20	[13]
FRDM analysis of atomic masses (2012)	32.5 ± 0.5	70 ± 15	[14]
IAS+LDM analysis of masses (2009)	32.5 ± 1.0	95 ± 19	[15]
Atomic masses and n-skin in an empirical approach (2012)	32.1	64 ± 5	[16]
IQMD analysis of isospin diffusion at 35 MeV/A (2010)	30.1	52 ± 0	[17]
IQMD analysis of isospin diffusion at 50 MeV/A (2009)	32.5 ± 2.5	77.5 ± 32.5	[18, 19]
IBUU04 analysis of isospin diffusion at 50 MeV/A (2005)	$30.5 \pm 0.$	86 ± 25	[20, 21]
isoscaling analysis of fragments (2007)	31.6	65	[22]
Liquid drop model analysis of atomic masses (2012)	$29.6 \pm 3.$	46.6 ± 37	[23]
Droplet Model+n-skin (2009)	31.5 ± 3.5	55 ± 25	[24, 25]
SHF+n-skin (2010)	30.5 ± 5.5	41 ± 41	[26]
Atomic masses (2010)	31.1 ± 1.7	66 ± 13	[27]
Global nucleon optical potential (2010)	31.3 ± 4.5	52.7 ± 22.5	[28]
Pygmy dipole resonances (2007)	32 ± 1.8	43 ± 15	[29]
Pygmy dipole resonances (2010)	32 ± 1.3	65 ± 16	[30]
Thomas-Fermi model analysis of masses (1996)	32.65	50	[31]
AMD analysis of transverse flow (2010)	30.5	65	[32]
r-mode instability of neutron stars (Vidana, 2012)	$30. \pm 5$	≥ 50	[33]
r-mode instability of neutron stars (Wen, 2012)	32.5 ± 7.5	≤ 65	[34]
Mass-radius of neutron stars-analysis1 (2010)	31 ± 3	50 ± 10	[35]
Mass-radius of neutron stars-analysis2 (2012)	33 ± 1.6	46 ± 10	[36]
Torsional crust oscillation of neutron stars (Gearheart, 2011)	32.5 ± 7.5	≤ 50	[37]
Torsional crust oscillation of neutron stars (Sotani, 2012)	32.5 ± 7.5	115 ± 15	[38]
Binding energy of neutron stars (2009)	32.5 ± 7.5	≤ 70	[39]

[5, 6, 46–48, 55–60]. Thus, a convincing conclusion on this issue will have profound ramifications in both nuclear physics and astrophysics. The momentum dependence of the isovector potential is conventionally measured by using the neutron-proton effective mass splitting

$$m^*_{n-p}(\rho_0, \delta) \equiv \frac{m^*_{\mathrm{n}} - m^*_{\mathrm{p}}}{m} \approx -2\delta \frac{m}{\hbar^2 k_{\mathrm{F}}} \frac{dU_{\mathrm{sym}}}{dk}\Big|_{k_{\mathrm{F}}} \Big/ (2\frac{m}{m^*_0} - 1). \tag{7}$$

According to Eq. 6, the $m^*_{n-p}(\rho_0, \delta)$ is completely determined by the $E_{sym}(\rho_0)$ and ł0 via

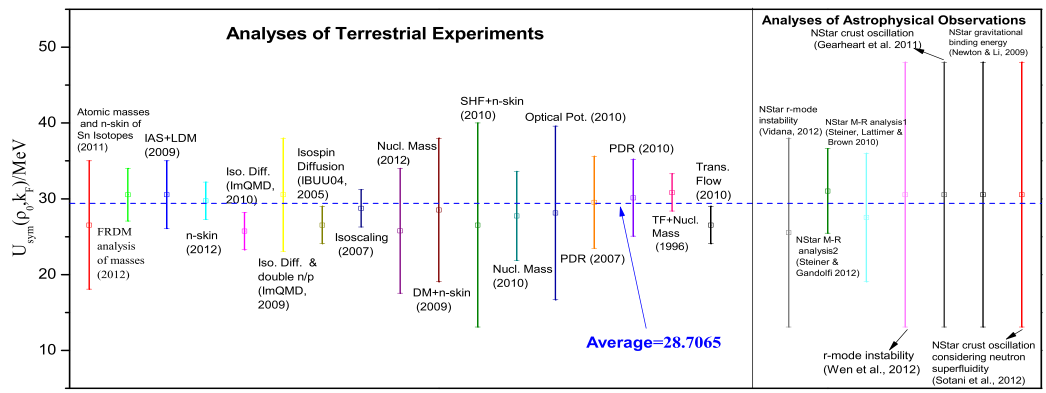
$$m^*_{n-p}(\rho_0, \delta) = \delta \cdot \left[3E_{\mathrm{sym}}(\rho_0) - L(\rho_0) - \frac{1}{3}\frac{m}{m^*_0}E_F(\rho_0)\right] \Big/ \left[E_F(\rho_0) \cdot (2\frac{m}{m^*_0} - 1)\right]. \tag{8}$$

It is clear that whether the m^*_{n} is equal, larger or smaller than the m^*_{p} depends on if the condition $L(\rho_0) \leq [3E_{\mathrm{sym}}(\rho_0) - \frac{1}{3}\frac{m}{m^*_0}E_F(\rho_0)]$ is satisfied. For example, using the most widely accepted empirical vales of $E_{sym}(\rho_0)$ =31 MeV, $m^*_0/m =$ 0.7 MeV and $E_F(\rho_0) = 36$ MeV, to obtain a $m^*_{n-p}(\rho_0, \delta) \geq 0$ a value of ł0$\leq$ 76 MeV is required.

III. NEUTRON-PROTON EFFECTIVE MASS SPLITTING FROM CONSTRAINS ON THE DENSITY DEPENDENCE OF NUCLEAR SYMMETRY ENERGY AROUND NORMAL DENSITY

It is known that essentially all proposed nuclear interactions have been used in various many-body theories to predict the $E_{sym}(\rho)$ [6]. Instead of using pure model prediction, we use here the $E_{sym}(\rho_0)$ and ł0 extracted from analyzing terrestrial nuclear laboratory experiments and astrophysical observations. Naturally, all analyses are based on some models and often different approaches are used in analyzing the same data or observations. For instance, at least 5 different models have been used to extract independently the $E_{sym}(\rho_0)$ and ł0 from studying atomic masse. Remarkably, however, with very few exceptions, constraints on the $E_{sym}(\rho_0)$ and ł0 from various analyses of the

same or different experiments/observations overlaps closely. Listed in Table 1 are 24 sets of constraints including the 4 astrophysical ones where only the upper or lower limit of ł0 is given. We notice here that while some of the reported constraints provide both the upper and lower limits or the standard deviation together with the mean values, some do not provide any information about the associated uncertainties. This situation will be carried over into calculating the corresponding $U_{\rm sym}(\rho_0, k_F)$ and $m^*_{n-p}(\rho_0, \delta)$. We also notice that the constraints on the $E_{sym}(\rho_0)$ and ł0 are treated here as quasi-data. Moreover, we use the most widely accepted empirical values for m^*_0/m and $E_F(\rho_0)$ in the formalism given in the previous section. Shown in Fig. 1 are the nucleon isovector potential (upper window)



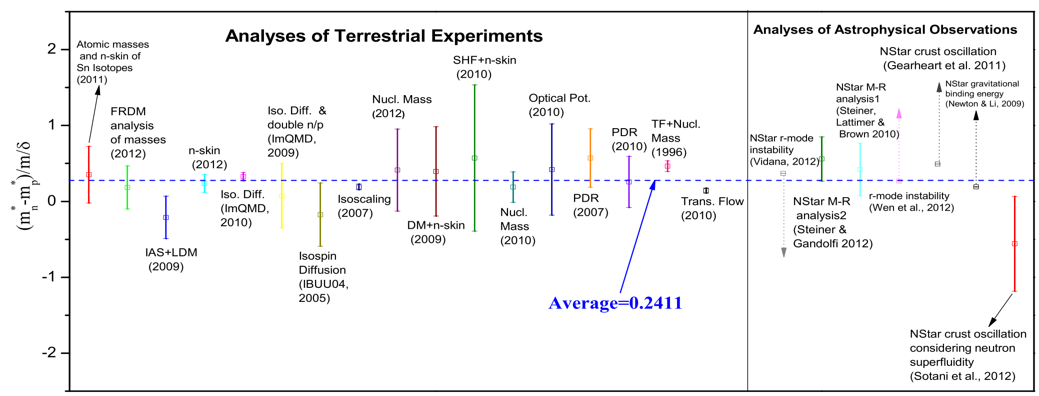


FIG. 1: (Color online) Nucleon isovector potential $U_{sym}(\rho_0, k_F)$ (upper) and neutron-proton effective mass splitting $m^*_{n-p}(\rho_0, \delta)/\delta$ (lower) at normal density of nuclear matter from 24 analyses of terrestrial nuclear laboratory experiments and astrophysical observations.

and neutron-proton effective mass splitting (lower window) at ρ_0 from the 24 constraints. We caution here that in cases where no error bar or range for the $E_{sym}(\rho_0)$ or ł0 was given, only the error bar of the empirical value of m^*_0/m is used in estimating the upper and lower limits of $U_{\rm sym}(\rho_0, k_F)$ and $m^*_{n-p}(\rho_0, \delta)$. In cases where only the upper or lower limits of ł0 were given, the limiting values were indicated with arrows for comparisons. It is interesting to see that despite of the large uncertainty ranges of some of the constraints on $E_{sym}(\rho_0)$ and ł0, the resulting mean values of $U_{\rm sym}(\rho_0, k_F)$ and $m^*_{n-p}(\rho_0, \delta)$ from different studies scatter very closely around their global averages of $U_{sym}(\rho_0, k_F)$ =29 MeV and $m^*_{n-p}(\rho_0, \delta) = 0.24 \cdot \delta$, respectively, indicating a high level of consistency of different studies. Moreover, the majority of the inferred $m^*_{n-p}(\rho_0, \delta)$ are positive. While the mean values of $U_{\rm sym}(\rho_0, k_F)$ and $m^*_{n-p}(\rho_0, \delta)$ are useful in their own rights, to use them reliably as a useful reference for calibrating nuclear many-body theories and much needed inputs for investigating many interesting issues in both nuclear physics and astrophysics,

the community should strive at reducing the experimental error bars and providing more complete information about the associated uncertainties in extracting the $E_{sym}(\rho_0)$ and ł0 using various models from the experimental data. In this regard, it is encouraging to note that some concerted efforts in this direction are under way by many people in both the experimental and theoretical communities investigating the density dependence of nuclear symmetry energy.

IV. CONCLUSION

Based on the Hugenholtz-Van Hove theorem, nuclear symmetry energy and the neutron-proton effective k-mass splitting are explicitly related to each other. Available constraints on the symmetry energy can be used to infer directly the poorly known but very important neutron-proton effective k-mass splitting in neutron-rich nucleonic matter. As an example, we have shown that the constraints on nuclear symmetry energy $E_{sym}(\rho_0)$ and its density slope ł0 at ρ_0 from 24 recent studies of terrestrial nuclear laboratory experiments and astrophysical observations indicate consistently that the nuclear isovector potential and neutron-proton effective k-mass splitting at ρ_0 are approximately $U_{sym}(\rho_0, k_F)$ =29 MeV and $m^*_{n-p}(\rho_0, \delta) = 0.24 \cdot \delta$, respectively. The need of reducing experimental error bars and proving more information about the associated uncertainties in extracting the $E_{sym}(\rho_0)$ and ł0 are emphasized.

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